

Roll No.

BCA-205(N)

B. C. A. (Second Semester) EXAMINATION, May, 2017

(New Course)

Paper Fifth

MATHEMATICS-II

Time : Three Hours] [Maximum Marks : 75

Note : Section A is compulsory. Attempt any *seven* questions from Section B and any *one* question from Section C.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

8 each

1. (a) Discuss the maxima or minima of the function :

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

- (b) Find the range of the function $f(x) = \frac{1}{2 - \cos 3x}$.

2. (a) If $u = \{1 - 2xy + y^2\}^{-1/2}$, prove that :

$$\frac{\partial}{\partial x} \left\{ (1 - x^2) \frac{\partial u}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ y^2 \frac{\partial u}{\partial y} \right\} = 0$$

- (b) Using distance formula show that the points A (3, -2, 4), B (1, 1, 1) and C (-1, 4, -2) are collinear.

Section—B

6 each

3. Change the order of integration $\int_0^\infty \int_0^x e^{-x^2/y} dx dy$ and evaluate.
4. For any set A and B, show that $(A - B) = (A \cap B')$.
5. In group of 52 persons, 16 drink tea but not coffee and 33 drink tea :
- (a) How many drinks tea and coffee both ?
- (b) How many drinks coffee but not tea ?
6. Find the shortest distance between two lines :

$$\frac{x}{5} = \frac{y-2}{2} = \frac{z-3}{3}$$

and $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$

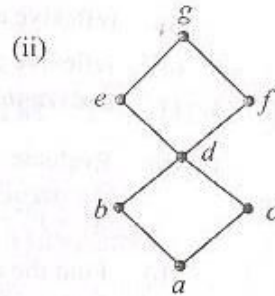
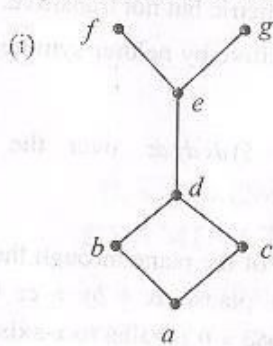
7. (a) Evaluate the following double integral :

$$\int_2^3 \int_0^1 (x^2 - 2y^2) dx dy$$

- (b) If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u$$

8. Determine whether the following Hasse diagrams represent lattice or not :



9. Let $f = \{(1, -1), (4, -2), (9, -3), (16, 4)\}$, $g = \{(-1, -2), (-2, -4), (-3, -6), (4, 8)\}$, show that $g \circ f$ is defined while $f \circ g$ is not defined. Also find $g \circ f$.

10. If $u = f(r)$, where $r^2 = x^2 + y^2$, then prove that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

11. Find the equation of the sphere with its center at $(3, 6, -4)$ and tangent in the plane $2x - 2y - z = 10$.

12. (a) If $f(x) = x^2 - \frac{1}{x^2}$, show that $f(x) + f\left(\frac{1}{x}\right) = 0$.

- (b) Draw the Hasse diagram of $(\Lambda \leq)$, where $\Lambda = \{3, 4, 12, 24, 48, 72\}$ and relation $\leq$ be such that $a \leq b$ if a divides b .

Section—C

17 each

13. Give an example of relation which is :

- (a) reflexive and transitive but not symmetric.

- (b) symmetric and transitive but not reflexive.
- (c) reflexive and symmetric but not transitive.
- (d) reflexive and transitive by neither symmetric nor antisymmetric.

14. (a) Evaluate $\iiint (z^5 + 5) dx dy dz$ over the sphere $x^2 + y^2 + z^2 = 1$.

- (b) Find the equation of the plane through the line of intersection of the plane $ax + by + cz + d = 0$ and $\alpha x + \beta y + \gamma z + \delta = 0$ parallel to x-axis.

15. (a) If a variable line in two adjacent position has direction cosine l, m, n and $l + \delta l, m + \delta m, n + \delta n$. Show that the small angle $\delta\theta$ between the two positions is given by :

$$(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$$

- (b) Find by double integration the area enclosed by the pair of curves $y = 2 - x$ and $y^2 = 2(2 - x)$.